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Numerical Models can be considered either simulations (attempts to reproduce reality) or experiments (attempts to learn something about a controlled process, *not necessarily physically realizable*).

It is generally held that:

“ Nobody believes a the results of a numerical model except the modeler,
while everyone believes an experiment except the experimentalist”
(a reworking of an A. Einstein quote)

V,V and UQ argues against this.

The fundamental cause of trouble in the world today is that the stupid are cocksure while the intelligent are full of doubt. – Bertrand Russell
(stolen from Carlo's email)

The less data you've collected the easier it is to understand.

Models of the solar convective envelope

The observational constraints – radiative and helioseismic signatures

What is being attempted?

What are the difficulties?

Focus on validation:

Are the observational constraints sufficient?

Which parts of the model are being validated by available observations?

ASH (Anelastic Spherical Harmonic):

- Global anelastic dynamics of an ideal gas in a rotating spherical shell

Mark Miesch (HAO/NCAR)

CSS (Compressible Spherical Segment):

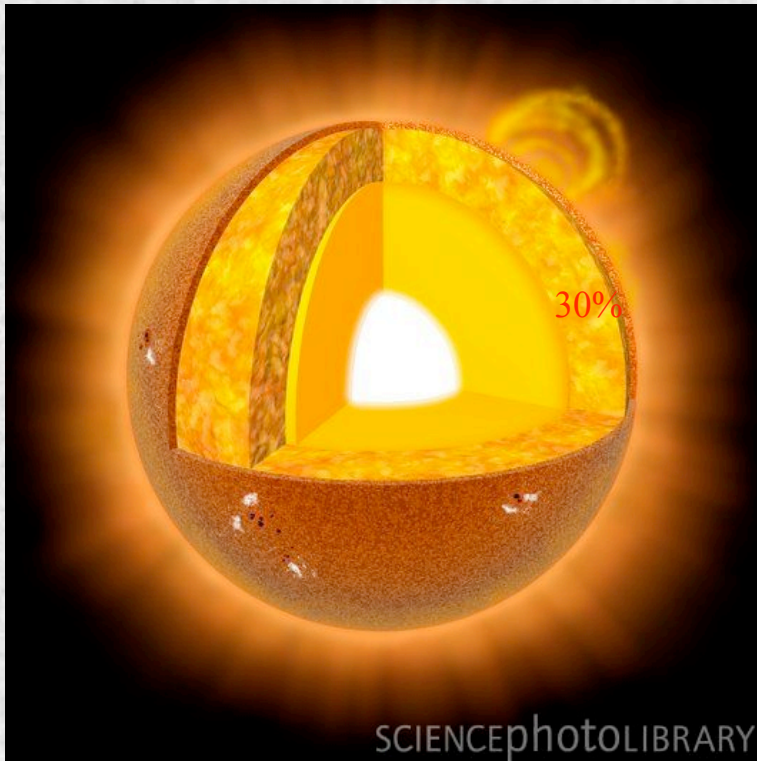
- Compressible dynamics of an ideal gas in a rotating shell segment

Kyle Augustson (University of Colorado, Boulder)

MSC (Modular Staggered Convection):

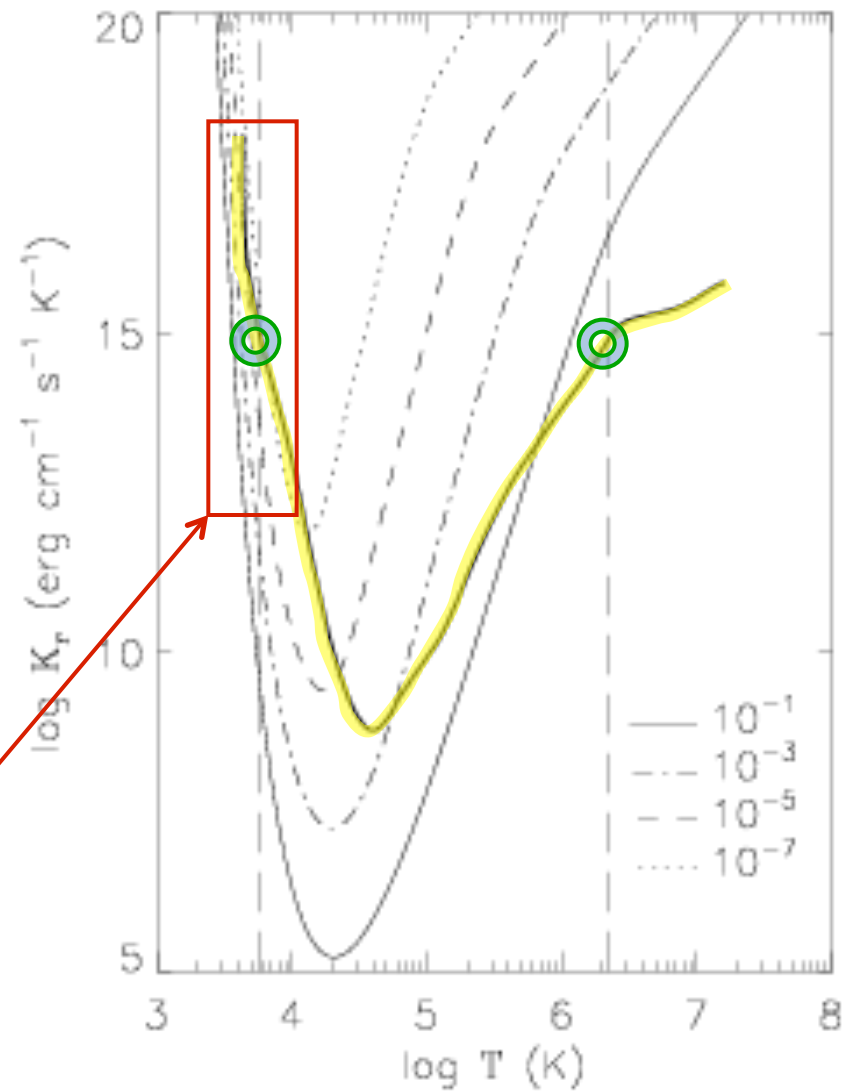
- Compressible dynamics of a plane parallel layer of solar plasma with radiative transfer

Åke Nordlund (University of Copenhagen) and Bob Stein (Michigan State)



$$K_r = \frac{16\sigma T^3}{3\bar{\kappa}\rho}$$

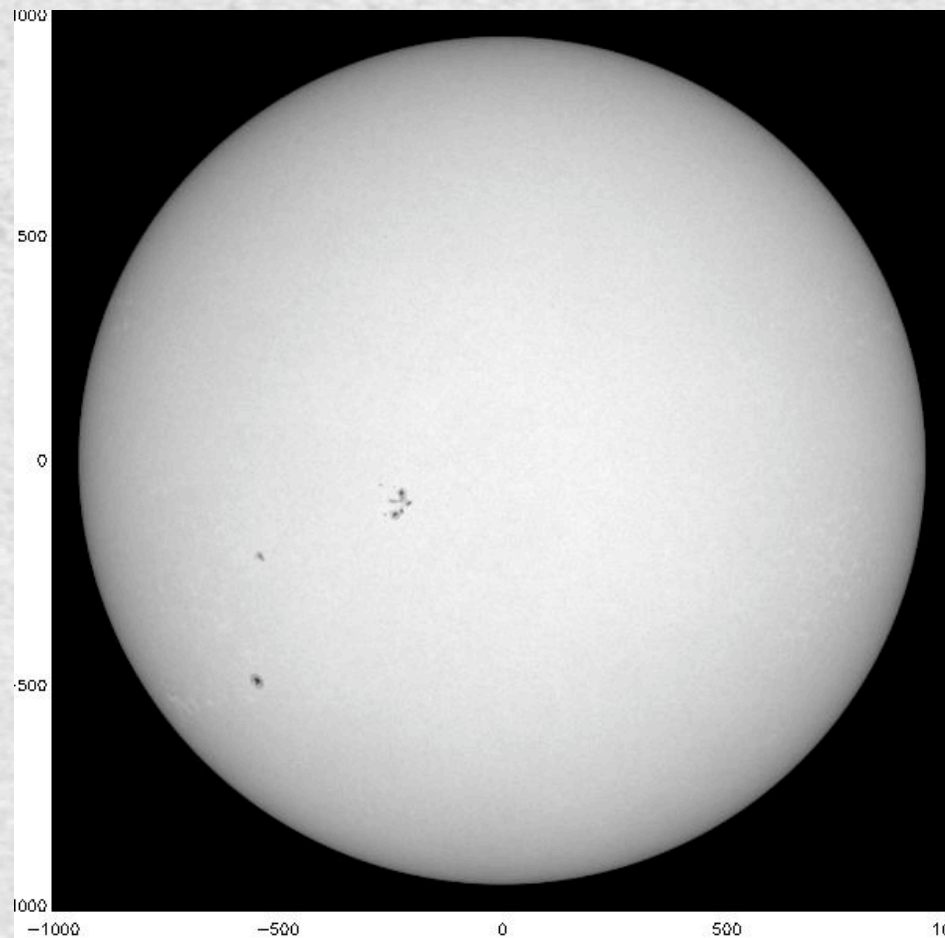
Rapid transition from convective transport to free streaming radiation



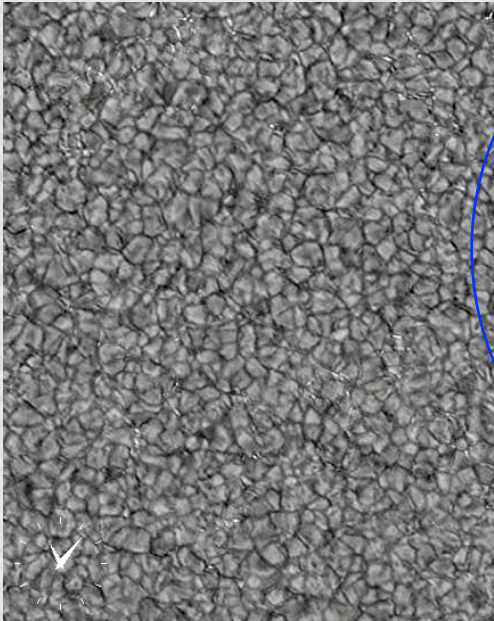
Observations:

Direct validation of models is confined to two possibilities:

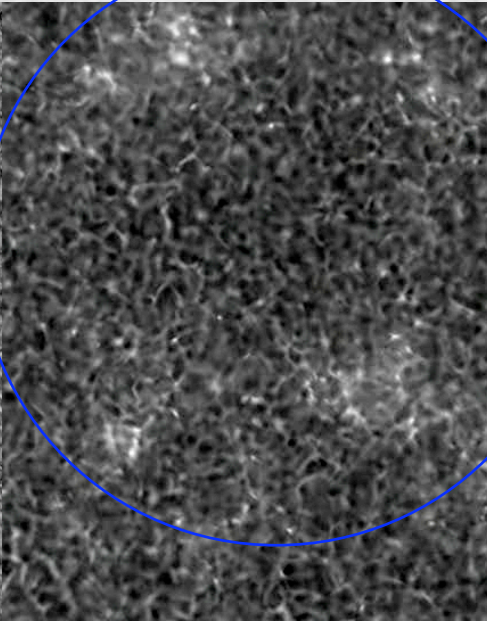
- Comparisons with observed properties of granulation in the photosphere
- Reproduction of global scale flows detected by helioseismology



G-band



CaIIH

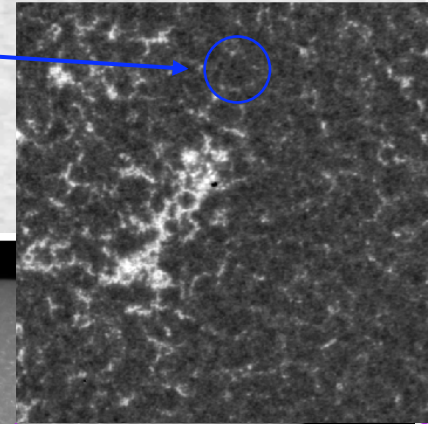


Supergranulation

(Hart 1954, Leighton et al. 1962)

- 32000km scale
- 400m/s horizontal flow
- 20hr lifetime

$Ro \sim 2$

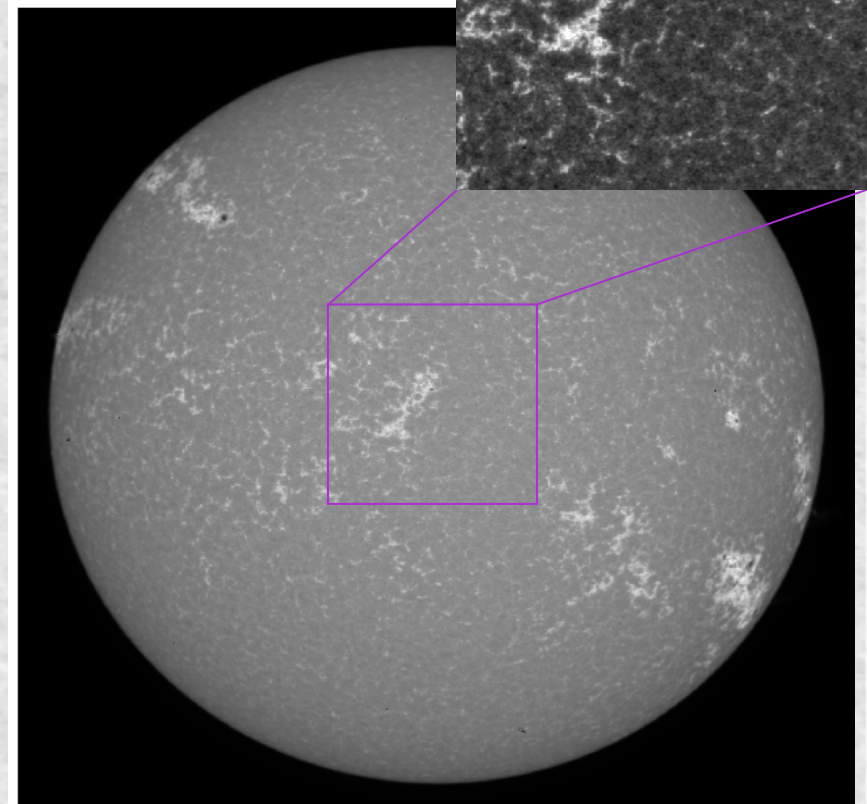


Dutch Open Telescope

Granulation (Herschel 1801)

- 1000km scale
- 1000m/s vertical flow
- 0.2hr lifetime

$Ro \sim 200$



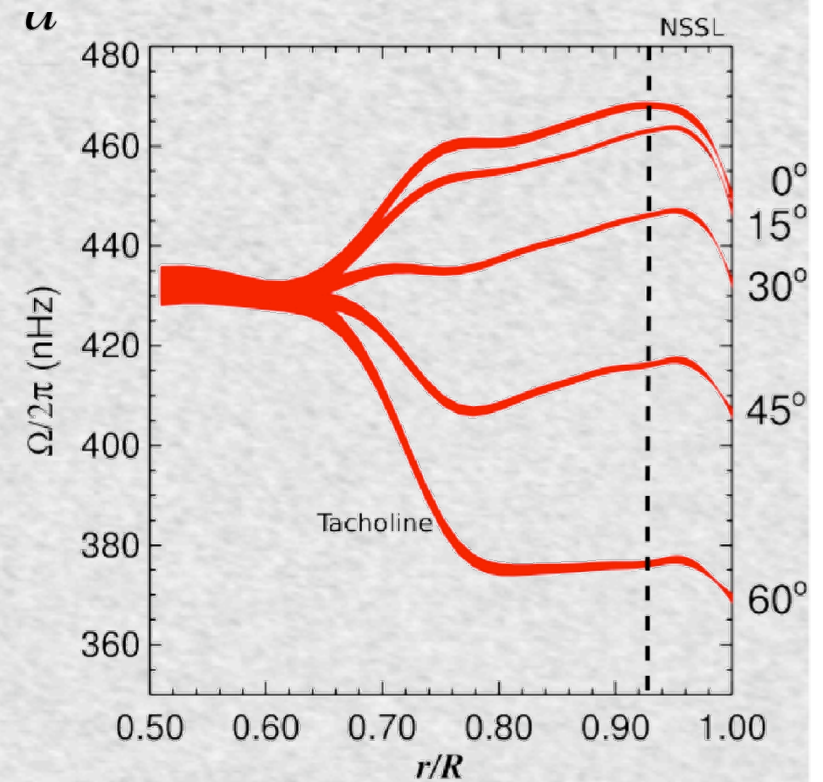
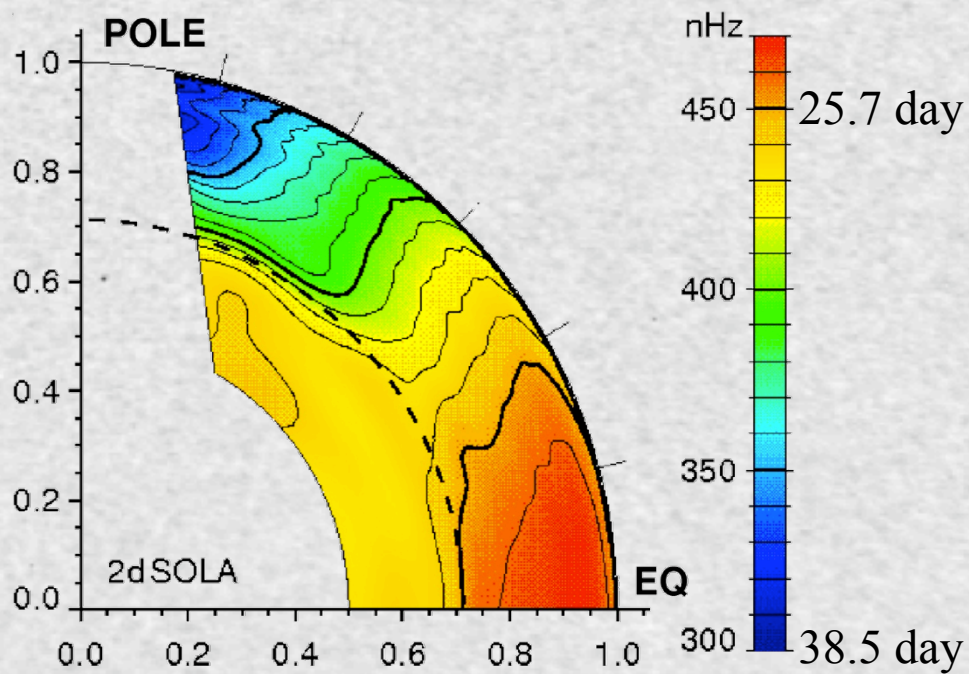
PSPT 20 March 2001 1731UT 393.5nm

Global scale motions

- Depth of convection zone, $\sim 200\text{Mm}$
- Transit time, $\sim 23\text{days}$
- Large scale convection (**giant cells**)?

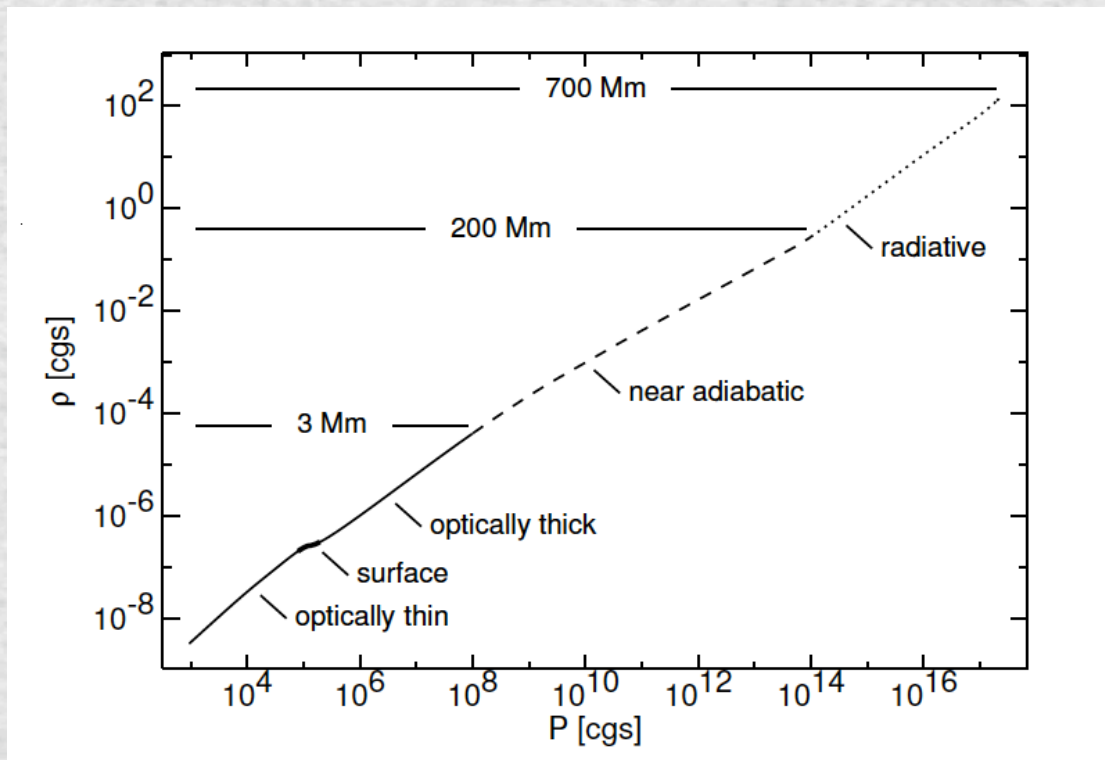
$Ro \sim 0.01$

Helioseismic global inversions



Modeling efforts span multiple physical domains:

- highly compressible radiative boundary layer
- region of partial hydrogen and helium ionization
- nearly adiabatic interior with 10^4 density contrast
- overshoot region at based of convection zone



Nordlund, Stein, and Asplund 2009

Solar convection experiments (numerical):

Solve the fully compressible or anelastic Navier-Stokes equations with

- hyperviscosity, slope limited diffusion, or subgrid model
- radiative transfer (LTE, 4 opacity bins) or thermal diffusion with subgrid model
- ideal gas eos or solar model table lookup

$$\begin{aligned}\frac{\partial \ln \rho}{\partial t} &= -\mathbf{u} \cdot \nabla \ln \rho - \nabla \cdot \mathbf{u} , \\ \frac{\partial \mathbf{u}}{\partial t} &= -\mathbf{u} \cdot \nabla \mathbf{u} + \mathbf{g} - \frac{P}{\rho} \nabla \ln P + \frac{1}{\rho} \nabla \cdot \boldsymbol{\tau} , \\ \frac{\partial e}{\partial t} &= -\mathbf{u} \cdot \nabla e - \frac{P}{\rho} \nabla \cdot \mathbf{u} + Q_{\text{rad}} + Q_{\text{visc}}\end{aligned}$$

$$\begin{aligned}\nabla \cdot (\bar{\rho} \mathbf{v}) &= 0, \\ \bar{\rho} \frac{\partial \mathbf{v}}{\partial t} + \bar{\rho} (\mathbf{v} \cdot \nabla) \mathbf{v} &= \\ &= -\nabla P - \rho g \hat{\mathbf{r}} - 2\bar{\rho}(\boldsymbol{\Omega}_* \times \mathbf{v}) - \nabla \cdot \mathbf{D} - \hat{\mathbf{r}} \left(\frac{d\bar{P}}{dr} + \bar{\rho} g \right) \\ \bar{\rho} \bar{T} \left(\frac{\partial S}{\partial t} + \mathbf{v} \cdot \nabla S \right) &= -\bar{\rho} \bar{T} v_r \frac{d\bar{S}}{dr} \\ &+ \nabla \cdot \left[\kappa_r \bar{\rho} C_P \nabla (T + \bar{T}) + \kappa \bar{\rho} \bar{T} \nabla S + \kappa_0 \bar{\rho} \bar{T} \frac{d\bar{S}}{dr} \hat{\mathbf{r}} \right] + \bar{\phi}.\end{aligned}$$

Codes span different domains of applicability – Code boundaries are problematic.

Modular Staggered Convection

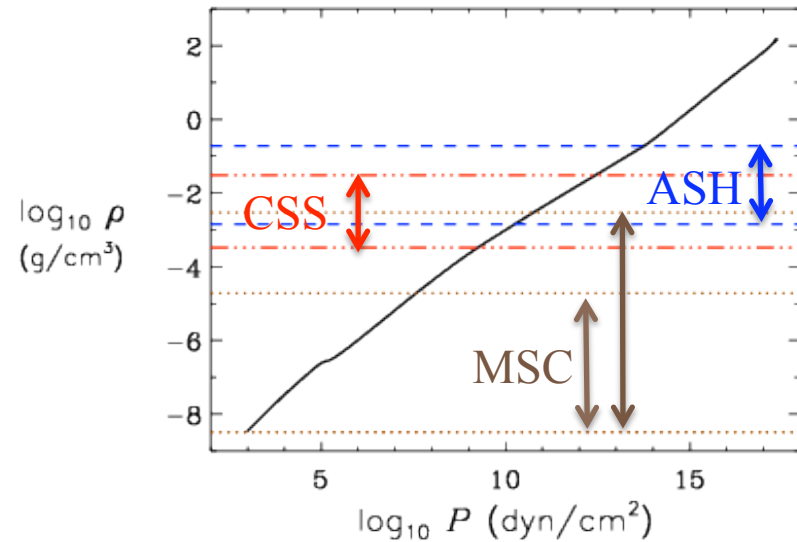
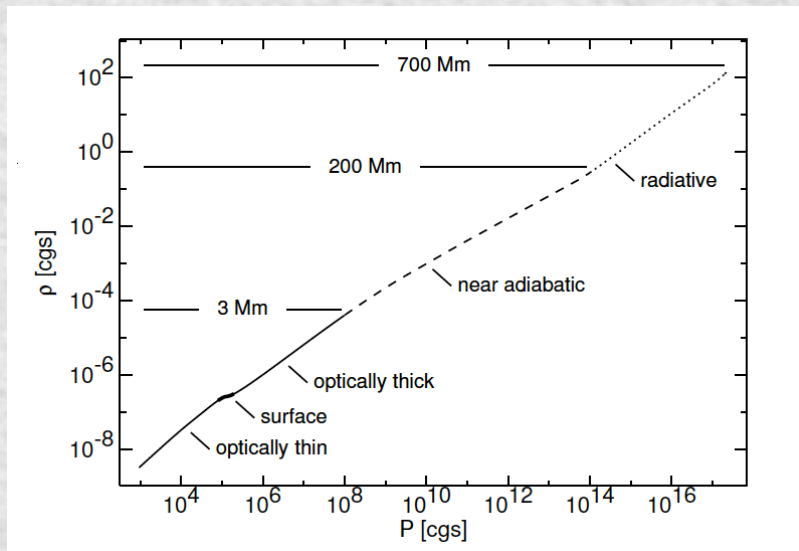
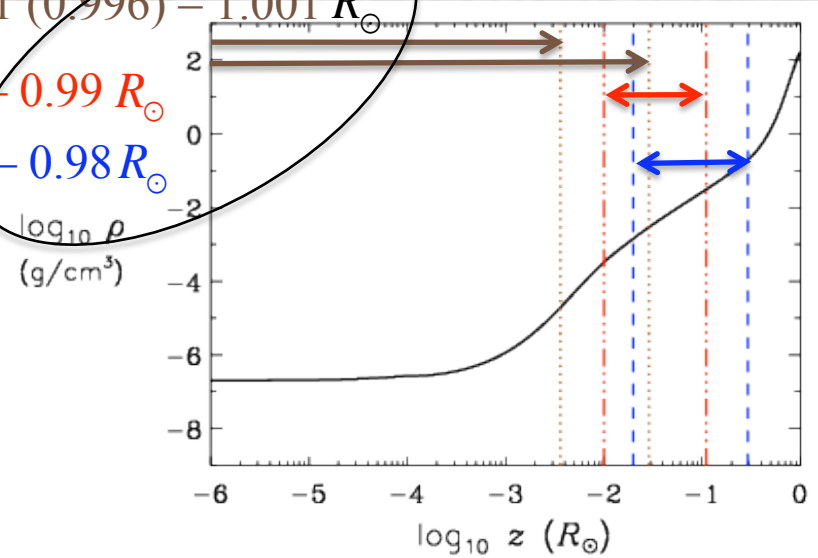
Compressible Spherical Segment

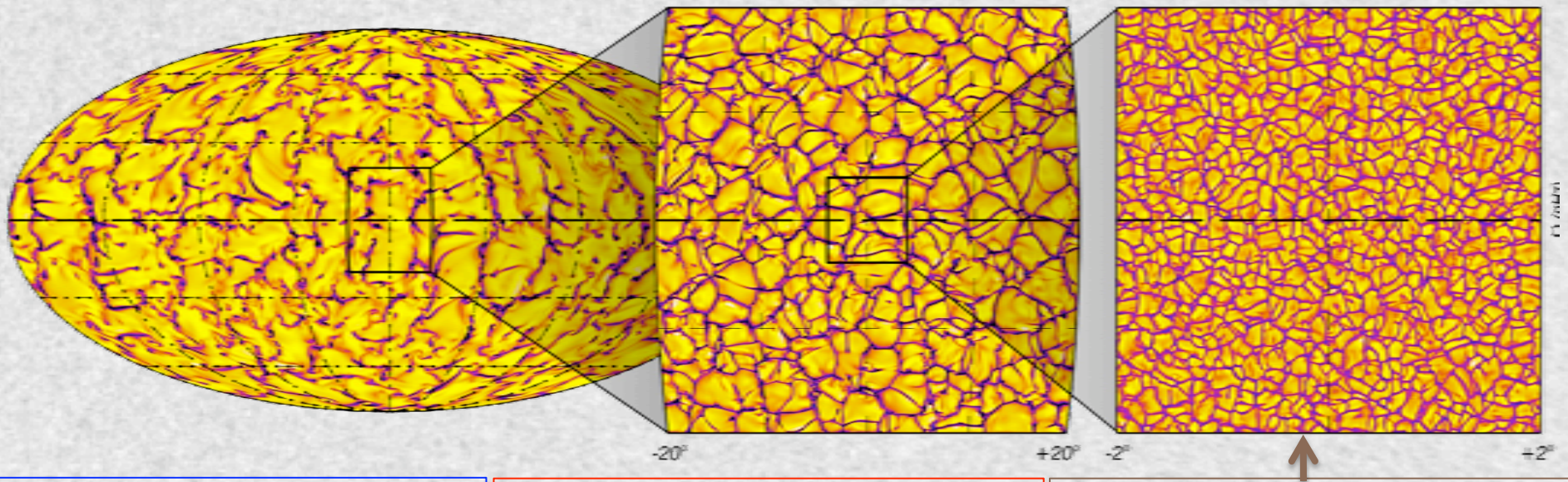
Anelastic Spherical Harmonic

MSC: $0.971 (0.996) - 1.001 R_{\odot}$

CSS: $0.89 - 0.99 R_{\odot}$

ASH: $0.71 - 0.98 R_{\odot}$

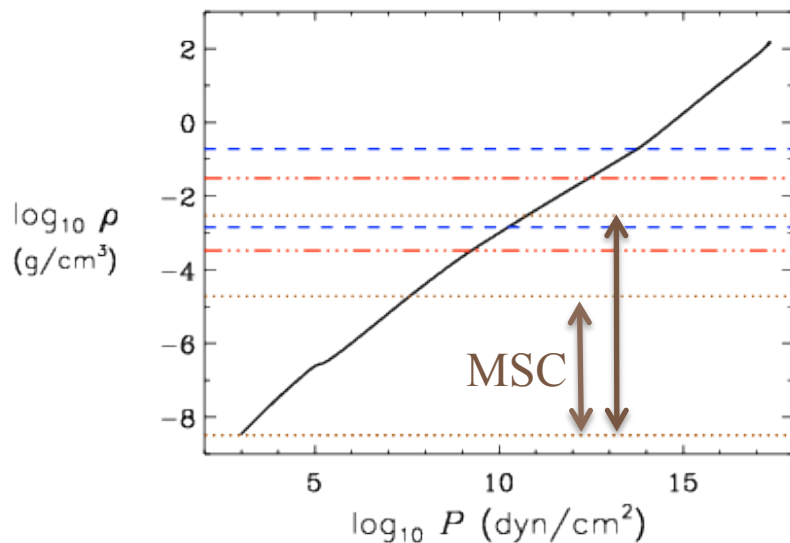




ASH (Anelastic Spherical Harmonic):

- Global anelastic ideal gas in a shell
- Resolution 257x1024x257 latitudinal, longitudinal, and vertical
- Stress free upper and lower boundaries
- Lower boundary includes latitudinal gradient (Reynolds stress)
- Upper boundary (region) includes enhanced unresolved radial flux

CSS (Compressible Spherical Segment):



of an ideal
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difference
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atitudinally
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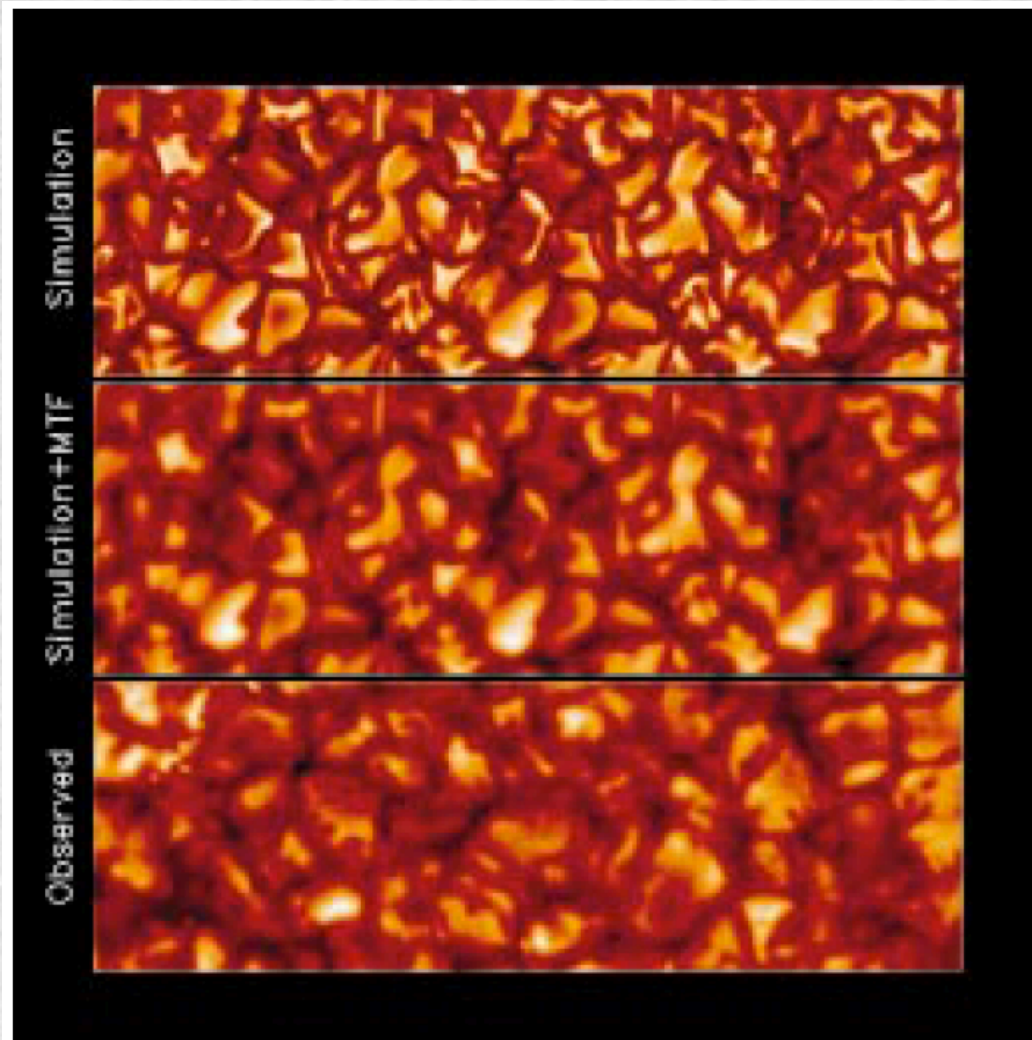
jets (mimicking statistics of down-
flows in high resolution granulation
simulations)

MSC (Modular Staggered Convection):

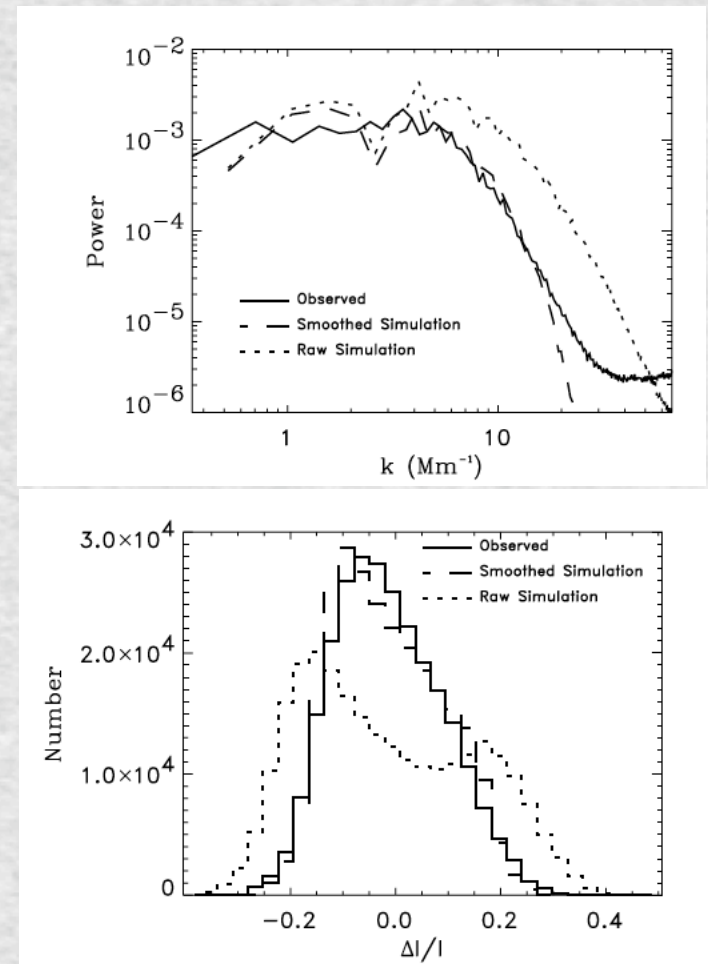
- Compressible dynamics of a plane parallel layer of solar plasma with radiative transfer
- 6th order compact finite differences, 3rd order Runge-Kutta time-stepping scheme
- Horizontally periodic, open upper and lower boundaries – leave outflows unaffected
- Lower boundary: entropy of inflows specified to meet solar flux, lower surface constant pressure

Simulation vs. Observations (MSC):

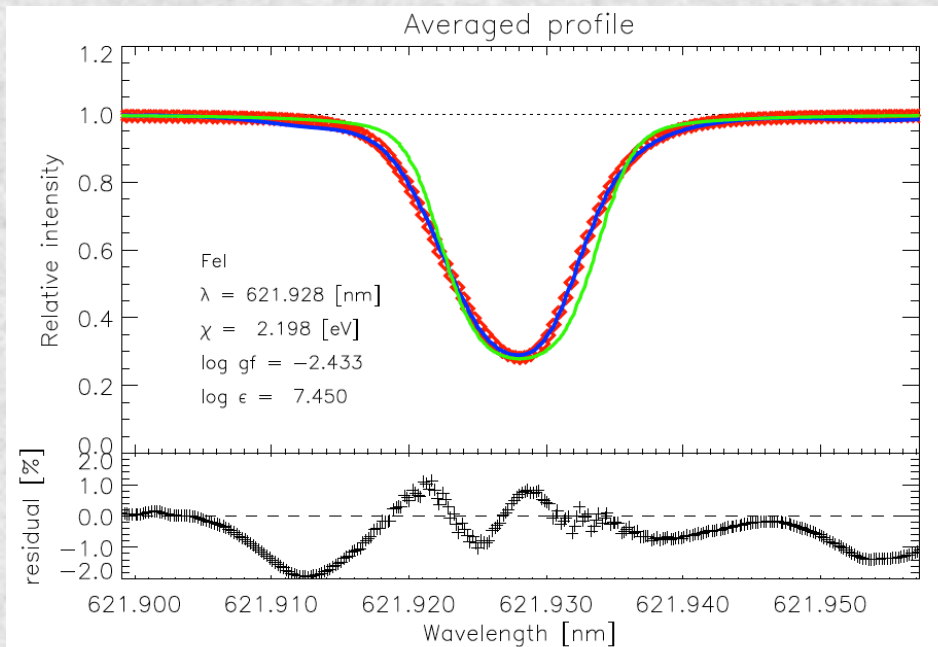
Stein and Nordlund 2000, Nordlund, Stein, and Asplund 2009



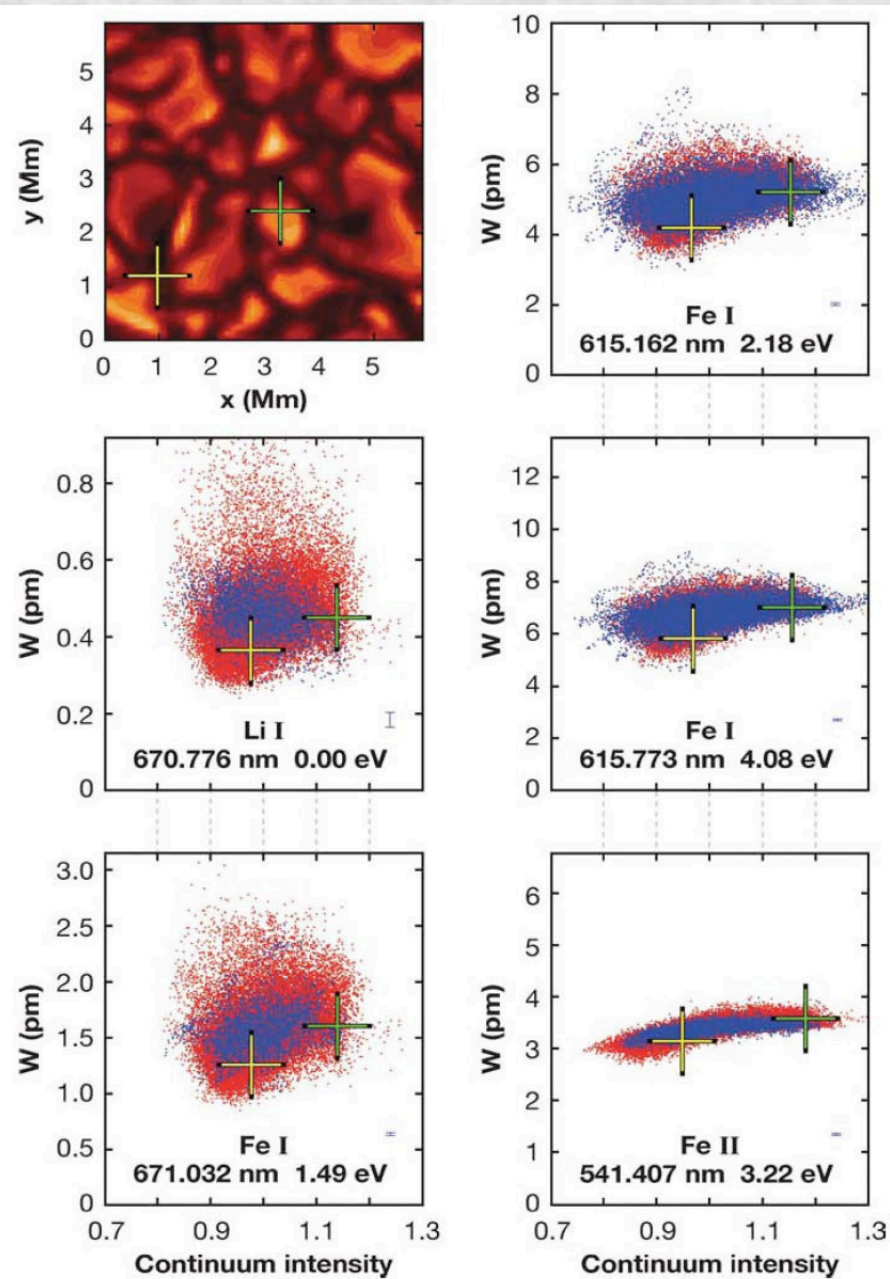
Viewgraph norm



Granular flow statistics and dynamics in good qualitative agreement



Spectral line properties compared
with solar disk center measurements



What physical processes in the code have been validated?

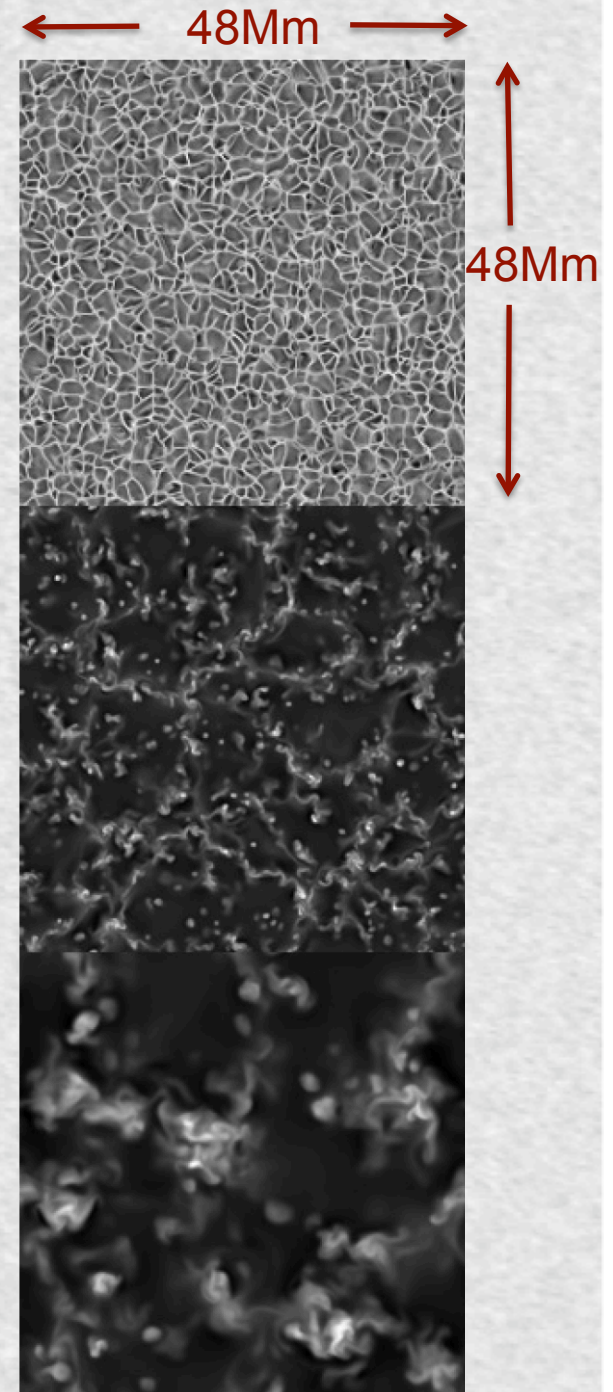
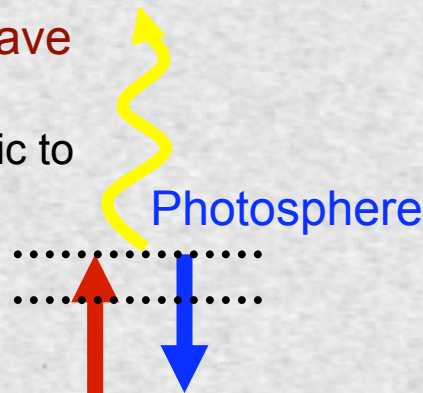
Observed flow properties may be generic to a radiatively cooled boundary layer of solar composition

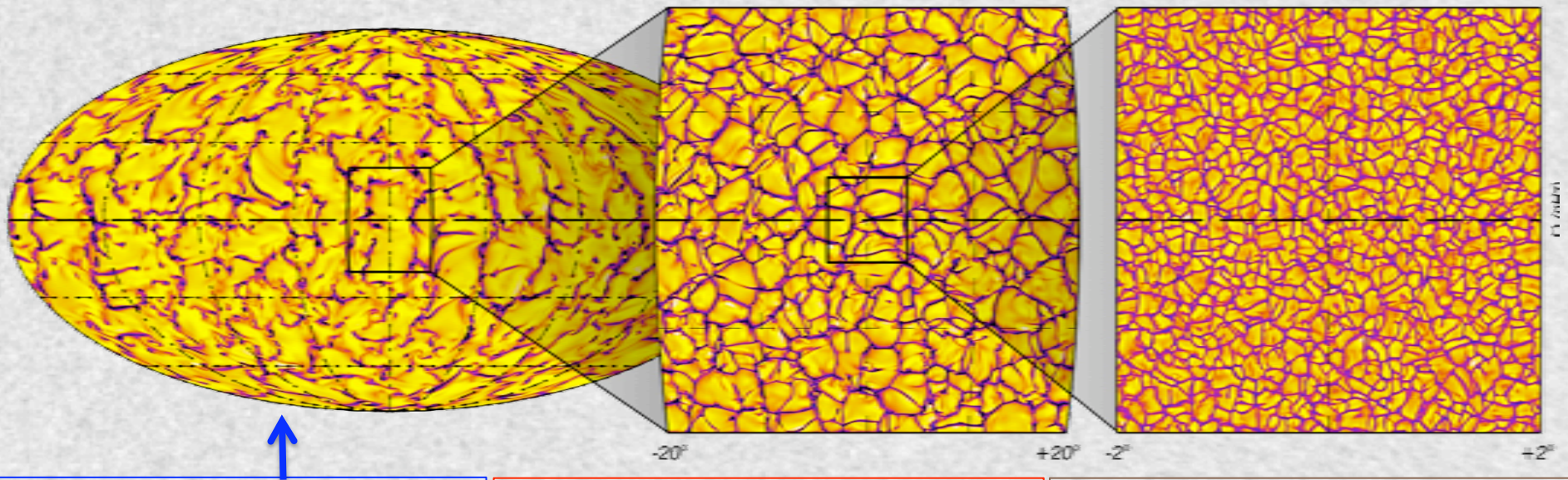
$$\sigma T^4 = \Delta h \rho w \implies w \approx 1.5 \text{ km/s}$$

Granulation dynamics is largely downflow dominated

The combination of a strongly radiatively cooled photosphere and an open lower boundary boundary that leaves downflows untouched and adjusts the upflows in response determines the solution characteristics (top-down dynamics)

The code is tailored to granulation but can not investigate the origin of larger scale motions in the photosphere if those have a bottom-up causes





ASH (Anelastic Spherical Harmonic):

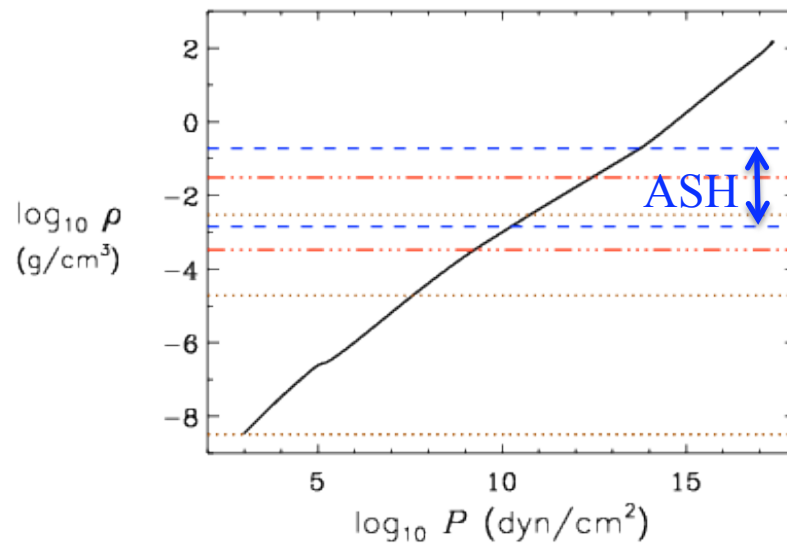
- Global anelastic dynamics of an ideal gas in a rotating spherical shell
- Resolutions up to 257x1024x2048 (radial, latitudinal, longitudinal)
- Stress free and impenetrable upper and lower boundaries
- Lower boundary condition includes latitudinal entropy gradient (Rempel 2005)
- Upper boundary (region) includes enhanced unresolved radial flux

CSS (Compressible Spherical Segment):

- Compressible gas in a rotating spherical segment
- 6th order spatial discretization, Runge-Kutta
- Longitudinal stress free, open or closed boundaries
- Stochastic boundary conditions (thermal fluctuations, jets (mimicking statistics of down-flows in high resolution granulation simulations))

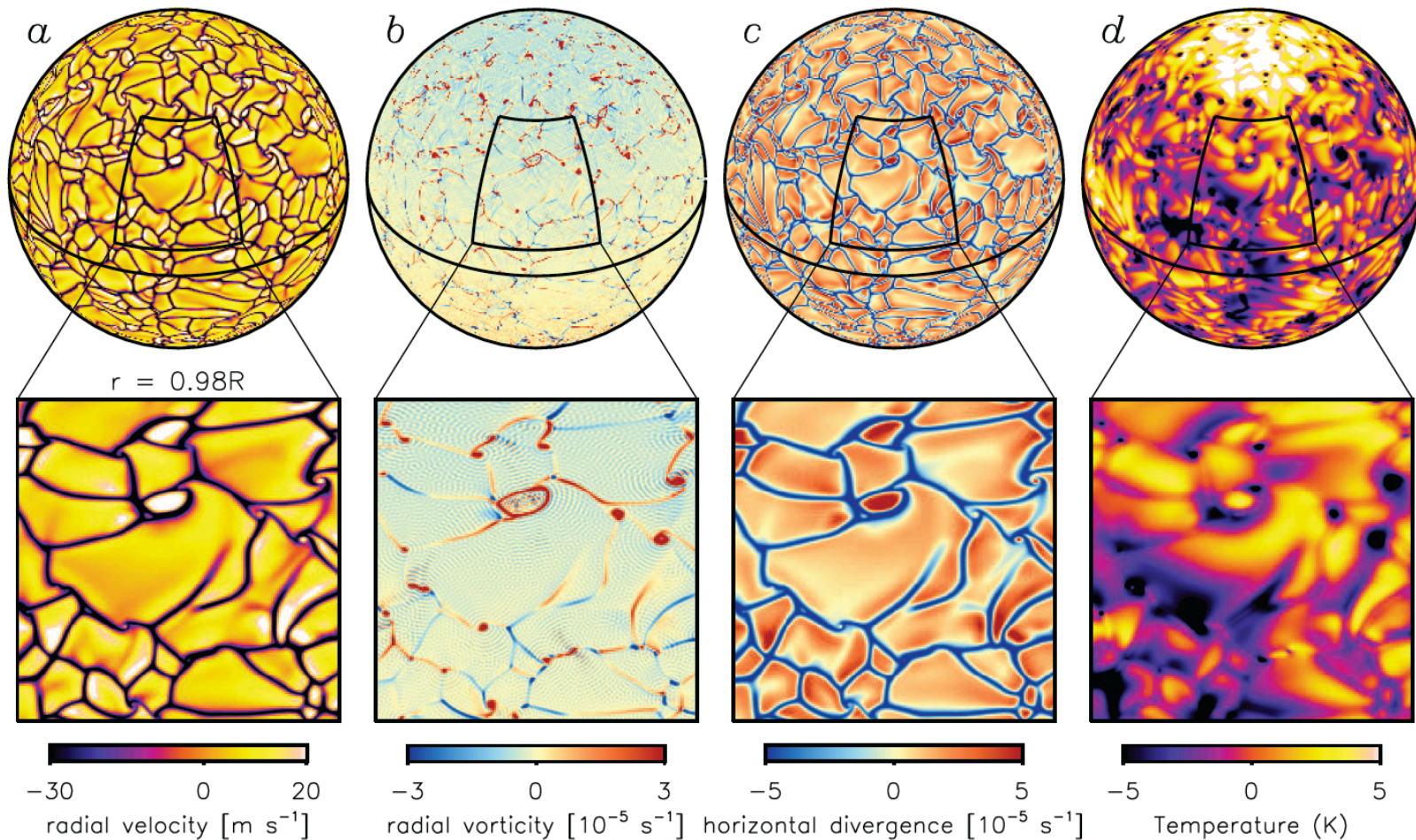
MSC (Modular Staggered Convection):

ASH is a global anelastic dynamics of a plane parallel plasma with periodic boundary conditions. MSC is a modular staggered convection scheme with periodic boundary conditions, open upper and lower boundaries – leave the upper boundary open to the entropy of the incoming solar flux, and the lower surface constant pressure.



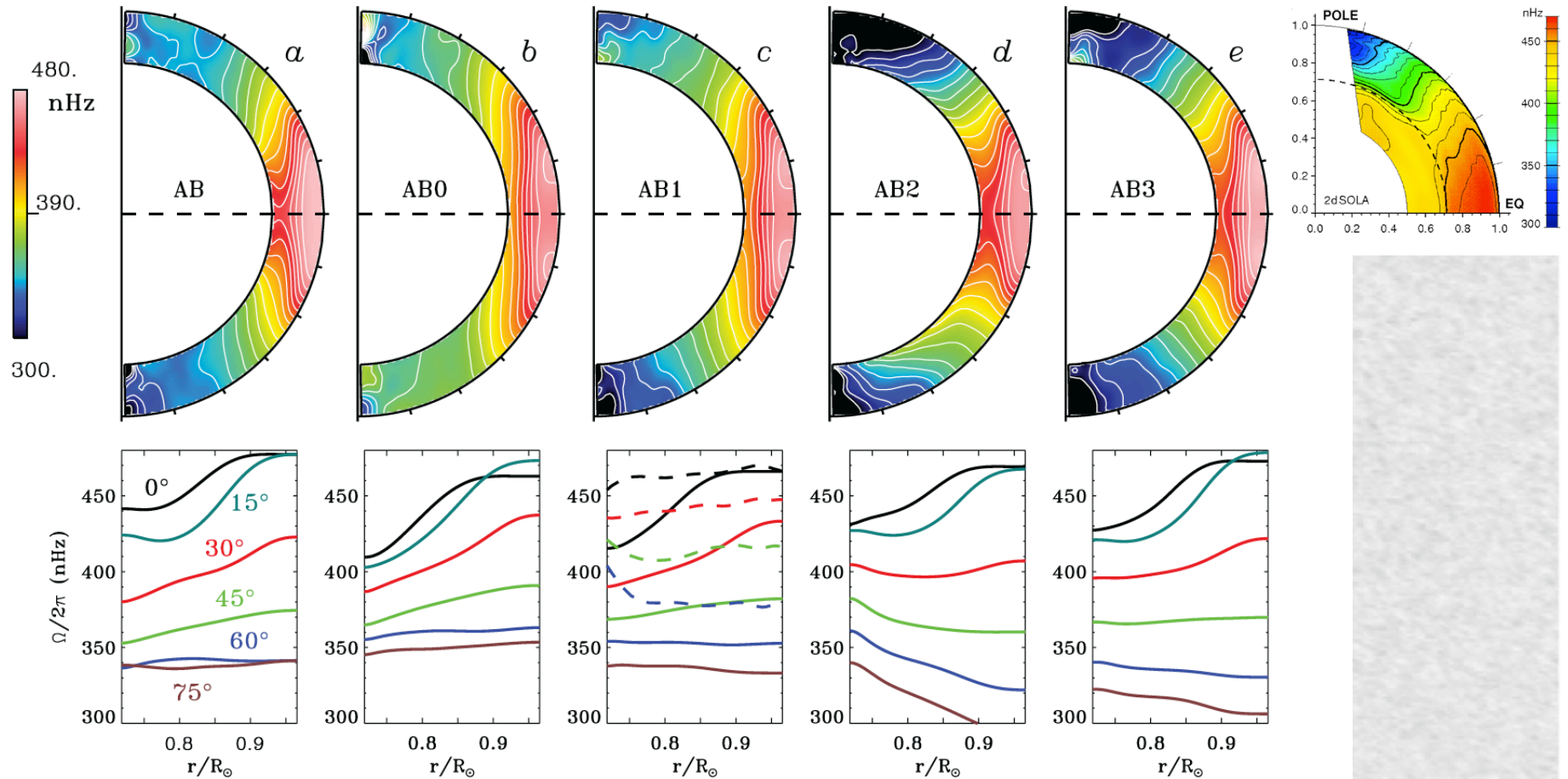
In ASH the opposite problem occurs:

At the top of the domain there is a thermal boundary layer that shouldn't be there



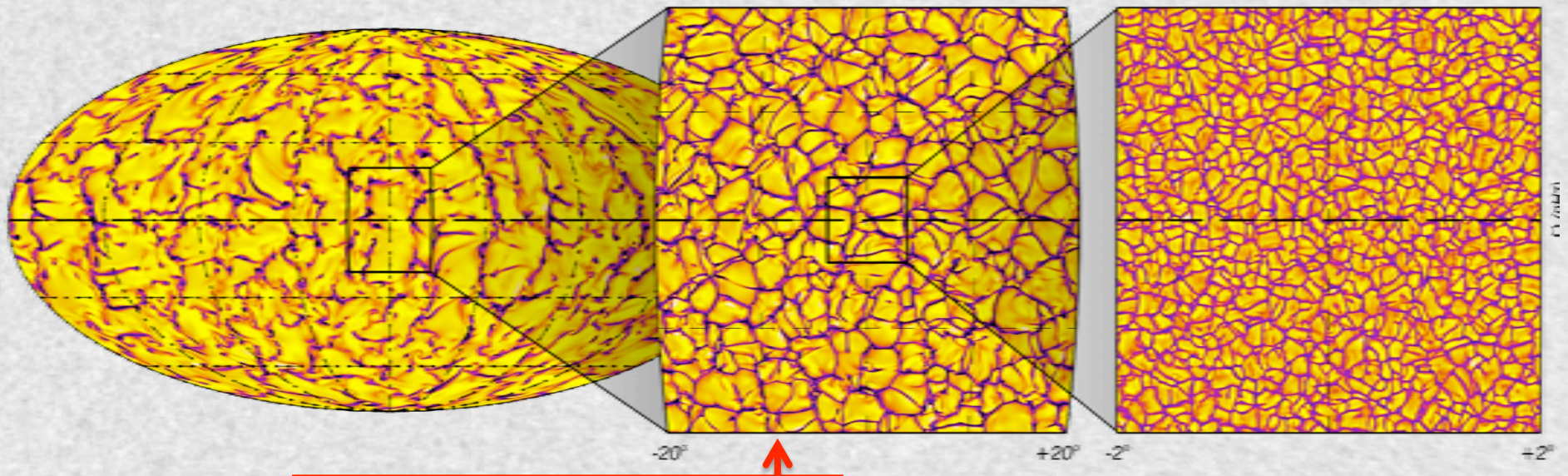
STRUCTURE AND EVOLUTION OF GIANT CELLS IN GLOBAL MODELS OF SOLAR CONVECTION
THE ASTROPHYSICAL JOURNAL, 673:557–575, 2008 January 20

Simulation vs. Observations (ASH):



Miesch, Brun, and Toomre 2006

- Imposed latitudinal entropy gradient at lower boundary improves differential rotation profile (Rempel 2005)
- Surface shear layer unattainable



ASH (Anelastic Spherical Harmonics)	CSS (Compressible Spherical Segment)	CSS (Compressible Spherical Segment)	MSC (Modular Staggered Convection):
<ul style="list-style-type: none"> Global anelastic dynamics of an ideal gas in a rotating spherical shell Resolutions up to 257x1024x2048 (radial, latitudinal, longitudinal) Stress free and impermeable upper and lower boundaries Lower boundary condition includes latitudinal entropy gradient (Rempel 2005) Upper boundary (region) includes enhanced unresolved radial flux 	<ul style="list-style-type: none"> Compressible dynamics of an ideal gas in a rotating spherical segment 6th order compact finite difference spatial discretization, 4th order Runge-Kutta time stepping scheme Longitudinally periodic, laterally stress free and impermeable, open or closed Stochastic driving of upper boundary possible (e.g. either sites (thermal states) or plumes (jets) (mimicking convection flows in high resolution granulation simulations)) 	<ul style="list-style-type: none"> Compressible dynamics of an ideal gas in a rotating spherical segment 6th order compact finite difference spatial discretization, 4th order Runge-Kutta time stepping scheme Longitudinally periodic, laterally stress free and impermeable, open or closed Stochastic driving of upper boundary possible (e.g. either sites (thermal states) or plumes (jets) (mimicking convection flows in high resolution granulation simulations)) 	

Bridging the local and global models with CSS:

- Drive the upper boundary with randomly placed granular downflow plumes

Three boundary conditions implemented:

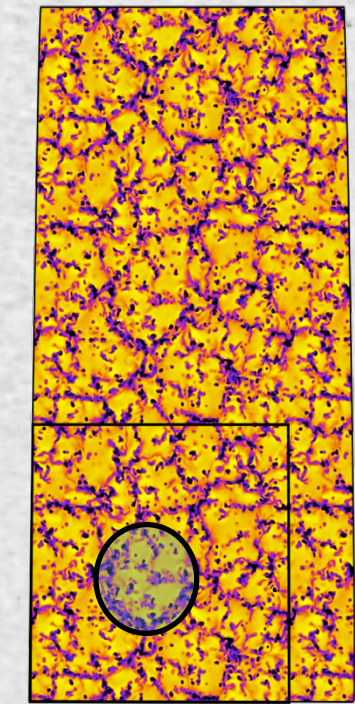
Spherically Symmetric Cooling – a sub-grid scale model of turbulent heat transport that diffusively carries flux through an impenetrable boundary (ASH like)

Starting Plumes – random cooling sites. The cooling strength of each plume scales with the total number of plumes, such that on average they carries the full luminosity, granular lifetimes

Thermal Jets – random inflows with a velocity, entropy deficit, duration, and size chosen from statistical distributions that mimic the downflow statistics in MSC

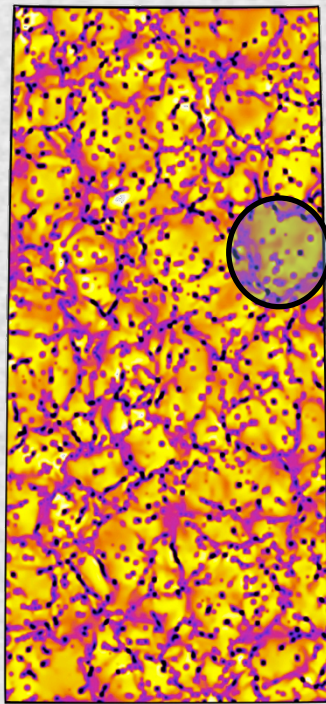
Horizontally averaged rotation rate:

Stein et al. (2009)

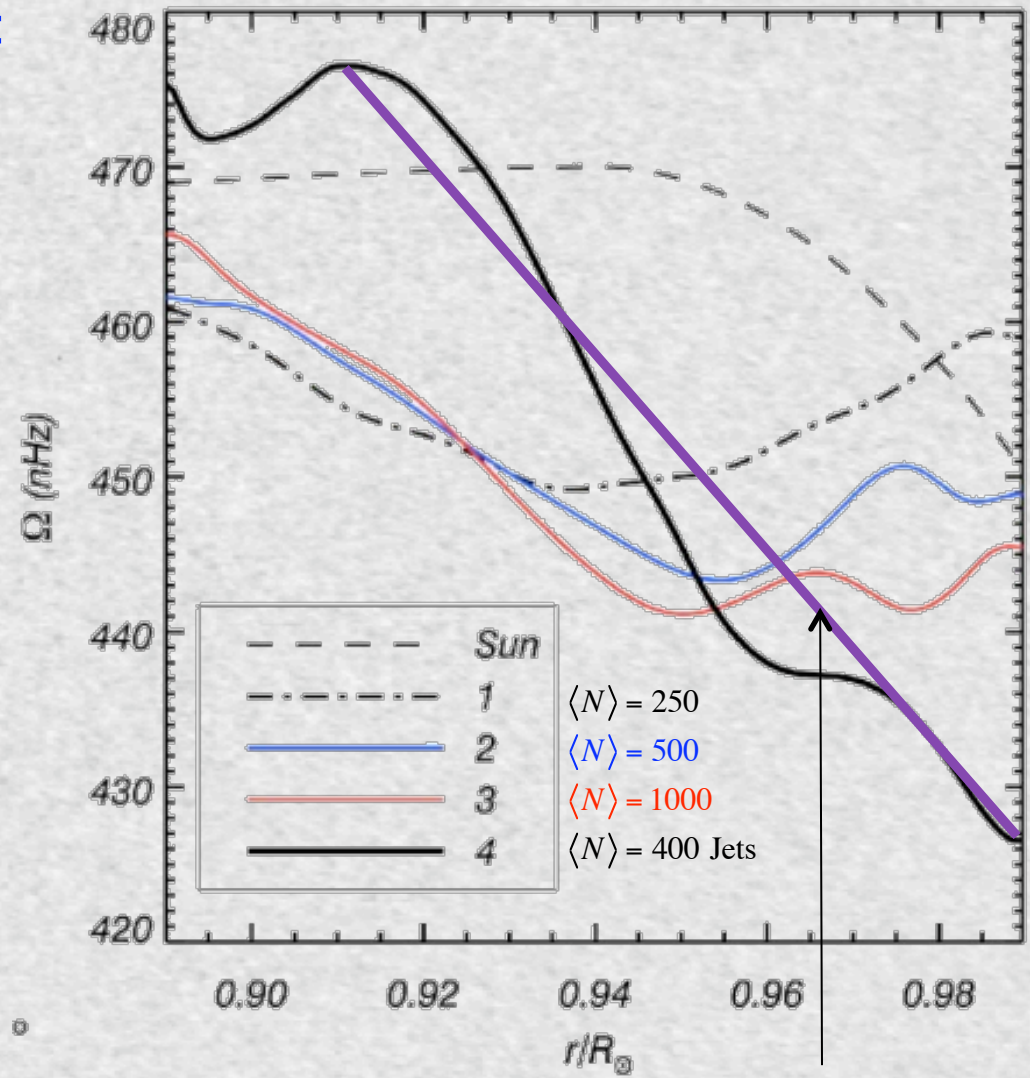


-5° 2500 x 1250 +5°

CSS:



-5° 512 x 256 +5°



Conservation
of specific
angular
momentum

Questions (which may display more my ignorance of v, v , and u_q than anything else):

1. How does one best conduct validation when the validation data is given by observations rather than focused experiments?

How does one assess which portions of the model are most constrained by the observations at hand, or how well constrained the overall model is by a limited set of observations?

2. What does validation mean in the context of numerical experiments?